Asset Pricing

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John H. Cochrane
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2001 Edition Back Flap:
John H. Cochrane is Sigmund E. Edelstone Professor of Finance at the University of Chicago Graduate School of Business and the author of many academic articles in financial economics. His work with John Campbell on investor behavior recently received the TIAA-CREF Paul A. Samuelson Certificate of Excellence. He is an editor of the *Journal of Political Economy*.

Note: In August 2006, John H. Cochrane is Myron S. Scholes Professor of Finance and a member of the board of directors of the Center for Research in Security Prices (CRSP) at Graduate School of Business, University of Chicago; formerly at the Department of Economics, University of Chicago.

Excerpts and Comments: 2001 Edition

Excerpt 1: Section 20.2 Subsection: The CAPM, page 436:

> In fact, Figure 20,8 captures one of the first significant *failures* of the CAPM. The smallest firms (the far right portfolios) seem to earn an average return a few percent too high given their betas. This is the celebrated "small-firm effect" (Banz [1981]). [Italics and quotation marks in original.]

Comment 1: The alleged small-firm effect or size effect is a serious violation of genuine method in the form of a fatal fallacy in logic known as vicious circular reasoning. In econometrics, it is called circular single-equation simultaneity. There are four instances in the Fama-French Three-Factor Model of expected total return for stock-portfolio pricing: in the same time period, the price variable is a component of return on the left-hand side and of both size and value on the right-hand side. Also in the same time period, the shares variable is a component of return on the left-hand side and of both size and value on the right-hand side. The alleged size effect is logically meaningless. The size factor is neither scientifically interesting nor important.

Excerpt 2: Section 20.2 Subsection: Fama-French 3 Factors, page 437:

> **Book market sorted portfolios show a large variation in average returns that is unrelated to market beta. The Fama and French three-factor model successfully explains the average returns of the 25 size and book market sorted portfolios with a three-factor model, consisting of the market, a small minus big (SMB) portfolio, and a high minus low (HML) portfolio.** [Bold-face text is in a framed insert.]
>
> The Fama-French model is one of the most popular multifactor models that now dominate empirical research. Fama and French (1993) presents the model; Fama and French (1996) gives an excellent summary, and also

shows how the three-factor model performs in evaluating expected return puzzles beyond the size and value effects that motivated it.

Figure 20.9 shows this value-size puzzle. … Figures 20.10 and 20.11 dig a little deeper to diagnose the problem.

Footnote 2: I thank Gene Fama for providing me with these data.

Comment 2: Book-to-market equity ratio, a/k/a book-to-market value ratio, includes size as its denominator. Therefore, like size, it is a serious violation of genuine method in the form of a fatal fallacy in logic known as vicious circular reasoning. In econometrics, it is called circular single-equation simultaneity. It is logically meaningless. It is neither scientifically interesting nor important.

Excerpt 3: Section 20.2 Subsection: What are the Size and Value Factors?, pages 441-443:

What are the macroeconomic risks for which the Fama-French factors are proxies or mimicking portfolios? There are hints of some sort of "distress" or "recession" factor at work.

A central part of the Fama-French model is the fact that these three pricing factors also explain a large part of the ex post variation in the 25 portfolios—the R^2 in time-series regressions are very high. In this sense, one can regard it as an APT rather than a macroeconomic factor model.

The Fama-French model is not a tautology, despite the fact that factors and test portfolios are based on the same set of characteristics.

[Bold-face text is in a framed insert.]

One way to assess whether the three factors proxy for real macroeconomic risks is by checking whether the multifactor model prices additional portfolios, and especially portfolios that do *not* have high R^2 values. Fama and French (1996) extend their analysis in this direction: They find that the SMB and HML portfolios comfortably explain

> strategies based on alternative price multiples (P/E, B/M). [Italics and quotation marks in original.]

Comment 3: The Fama-French size- and value-related factors are reverse proxies or backward proxies, and as such are logically meaningless. The unknown, unobservable, macroeconomic factors alleged to be behind the size and value factors are fantastical, mythical, intellectual speculation. Anything could be asserted to be the something for which the size and value factors are alleged to be proxies.

Furthermore, shares- and value-related factors, price-to-earnings ratio (P/E), and book-to-market equity ratio (B/M) are price- and shares-entailing factors. Thus, these price- and shares-entailing factors are alleged to explain the price- and shares-entailing expected returns. This is not an independent check or verification, but rather an example of vicious circular reasoning.

Wikipedia defines tautology, in logic, as "a statement true by virtue of its logical form"; in rhetoric, as "use of redundant language that adds no information"; and in general, as a truism, i.e., "an assertion that is so obvious as to add nothing to a discussion." Mr. Cochrane makes an inculpatory false statement when he writes: "The Fama-French model is not a tautology." On the contrary, the Fama-French Three-Factor Model of return is a tautology in all three senses of the term.

The price- and shares-entailing size and value alleged explanatory factors in the Fama-French Three-Factor model of price- and shares-entailing return are logical tautologies, because they and the dependent variable, expected total return, entail the price and shares variables in a form that makes the model necessarily true by definition and adds no new information. In classical logic, these size- and value-related alleged

return factors commit the same fatal fallacy of vicious, inferential, circular reasoning, in contrast to virtuous, non-inferential, circular reasoning, at both the theoretical level and the empirical level. The size- and value-related tautologies in the Three-Factor Model of return are fatal fallacies known in econometrics as logically circular single-equation simultaneities, in contrast to algebraically isolated single-equation simultaneities.

The logical invalidity of tautology and the fallacy in logic known as vicious circular reasoning share a common essential element: neither one adds any new information to an argument. Genuine method proceeds from the better known to the lesser known – it does not proceed from the known to the equally known. If something is equally known, it adds no new information. Tautology and vicious circular reasoning are both serious violations of genuine method. At the level of genuine method, both tautology and vicious circular reasoning are logically equivalent fatal fallacies that invalidate logical inference.

The return, size and value variables are decomposable into atomic formulas or primitive components (constants, variables, or functional terms). Unobservable total return *by definition* is derived from and therefore decomposable into five observable, non-decomposable components: beginning price, ending price, beginning shares, ending shares, and dividends. Unobservable size *by definition* is derived from and therefore decomposable into beginning price and beginning shares. Unobservable value *by definition* is derived from and therefore decomposable into book equity and size, which in turn is decomposable into beginning price and beginning shares. The Fama-French Three-Factor Model alleges in four instances that "$A = A$", "A is caused, determined or explained by A", "A precedes A", "if A, then A", and "then A, if A", where A symbolizes

either beginning price or beginning shares embedded in the size and value factors and in return.

Econometrics is a method of causal inference applied to economics. In an econometric model of expected total return for stock pricing or stock-portfolio pricing, any alleged explanatory factor that entails the price, shares or dividends variable as a component and occurs in the same time period as expected total return, results in logically circular single-equation simultaneity and is a fatal fallacy of vicious circular reasoning. There is no counter-example to this fallacy, and it ends a logical argument. In that sense, it is logically fatal. In addition, size *defined* as market equity and value *defined* as book equity-to-market equity ratio, are perfectly multicollinear, and thus their independent contributions to the explanation of return cannot possibly be disentangled. The statistical significance (Student's *t*-statistic test and *F* test) and the explanatory power (adjusted R^2 or adjusted coefficient of determination) are spuriously induced and meaningless for the Three-Factor Model of expected total return and its size- and value-related alleged explanatory factors.

Mr. Cochrane is silent about logically circular single-equation simultaneity in the Three-Factor Model of return, and he is silent about perfect multicollinearity in the Three-Factor Model of return. Mr. Cochrane knows or has reason to know that the Fama-French Three-Factor Model of return for stock-portfolio pricing is not logically valid, not econometrically valid, not economically valid, and not scientifically valid. This appears to be an extreme case of psychological denial, cognitive dissonance, or motivated irrationality.

In addition, Mr. Cochrane makes a disingenuous, misleading statement when he writes: “The Fama-French model is not a tautology, despite the fact that factors and test portfolios are based on the same set of characteristics.” The tautology invalidity and the vicious circular reasoning fatal fallacy occur in two instances: one, between the dependent variable and the alleged explanatory factors; two, between the dependent variable and the portfolio-formation variables. Using the term “characteristics” to refer to the alleged explanatory factors tends to mask their role in the econometric model.

These two instantiations involve three roles for the price-, shares- and dividends-entailing variables in the Fama-French Three-Factor Model of expected return for stock-portfolio pricing in both its 1992 and 1993 versions, as explained in the Appendix. The first role is the left-hand-side or dependent variable, whether individual individual-stock-based or stock-portfolio-based. The dependent variable, expected total return, entails five variables: beginning price, ending price, beginning shares, ending shares, and dividends. The left-hand-side variable is either stock-based return or portfolio-based return. The 25 portfolios for portfolio-based returns are the intersection of size-related quintile portfolios and value-related quintile portfolios.

The second role is the right-hand-side explanatory variable, whether individual-stock-based or stock-portfolio-based. Two of the three explanatory factors directly specified in the econometric model equation, size-related and value-related, entail the beginning price and beginning shares variables. The combination of the first and second roles results in four instantiations of direct circular single-equation simultaneity. These are four instantiations of the fatal fallacy of vicious circular reasoning and the logical invalidity of tautology, both of which are serious violations of genuine method.

The third role is the right-hand-side sample grouping variable, whether individual stock-based or stock portfolio-based. The two portfolio-formation factors, size (ME) and value (BE/ME), entail the beginning price and beginning shares variables. There are six size-BE/ME portfolios formed by the intersection of three size fractile portfolios and two BE/ME fractile portfolios. The combination of the first and third roles results in four instantiations of indirect circular single-equation simultaneity. These are four instantiations of the fatal fallacy of vicious circular reasoning and the logical invalidity of tautology, both of which are violations of genuine method.

The combination of the second and third roles is not vicious circular reasoning, which requires a variable for the same time or same time period to be on both the left-hand side and the right-hand side of the model equation simultaneously. The explanatory factors and the portfolio-formation factors in this case are both on the same side of the econometric model equation, the right-hand side. Yet, the combination of the second and third roles is a tautology in the sense of redundancy at the level of the size and value factors, before the individual stocks are grouped into portfolios with attendant loss of information. When both parts of this type of redundant combination occur as explanatory variables in the model equation, the result is direct perfect multicollinearity. When both parts occur as portfolio-formation variables, the result is indirect perfect multicollinearity. When one part occurs as an explanatory variable in the model equation and the other part occurs as a portfolio-formation variable, the result is hybrid direct-indirect perfect multicollinearity.

Excerpt 4: Section 20.2 Subsection: What are the Size and Value Factors?, pages 443-444:

One's first reaction may be that explaining portfolios sorted on the basis of size and book/market, by factors sorted on the same basis, is a tautology. This is not the case. For example, suppose that average returns were higher for stocks whose ticker symbols start later in the alphabet. (Maybe investors search for stocks alphabetically, so the later stocks are "overlooked.") This need not trouble us if Z stocks happened to have higher betas. If not—if letter of the alphabet were a CAPM anomaly like book/market—however, it would not necessarily follow that letter-based stock portfolios *move together*. Adding A-L and M-Z portfolios to the right-hand side of a regression of the 26 A, B, C, etc. portfolios on the market portfolio need not (and probably does not) increase R^2 at all. The size and book/market premia are hard to measure, and seem to have declined substantially in recent years. But even if they decline back to CAPM values, Fama and French will still have found a surprisingly large source of common movement in stock returns.

More to the point, in testing a model, it is exactly the right thing to do to sort stocks into portfolios based on characteristics related to expected returns. When Black, Jensen, and Scholes and Fama and MacBeth first tested the CAPM, they sorted stocks into portfolios based on betas, because betas are a good characteristic for sorting stocks into portfolios that have a large spread in average returns. If your portfolios have no spread in average returns—if you just choose 25 random portfolios, then there will be nothing for the asset pricing model to test.

In fact, despite the popularity of the Fama-French 25, there is really no fundamental reason to sort portfolios based on a two-way or larger sorts of individual characteristics. You should use all the characteristics at hand that (believably!) indicate high or low average returns and simply sort stocks according to a one-dimensional measure of expected returns.

The argument over the status of size and book/market factors continues, but the important point is that it does so. … Now we are back

> where we were, examining small anomalies and arguing over refinements and interpretations of the theory. [Italics and quotation marks in original.]

Comment 4: On the contrary, explaining expected return by size- and book/market-related explanatory factors, whether or not the factors are sorted on the same basis as expected return, is an invalid tautology and a fatal fallacy known in classical logic as vicious circular reasoning. This is true whether or not expected return is grouped to construct a portfolio-based dependent variable, and whether or not return fractile-portfolios are sorted on size and book/market factors. Price- and shares-entailing factors, such as size- and value-related factors, cannot logically or scientifically describe or explain price- and shares-entailing return. Such a causal, inferential, econometric model is a logically circular single-equation simultaneity that seriously deviates from generally accepted standards of scientific research methodological practice.

The argument of Mr. Cochrane is contradictory concerning the sorting of sample data and grouping of sample data for econometric model testing. On one hand, somewhere he correctly states that the sort order of the observations in a sample makes no difference regarding the parameter estimates and the statistical tests of significance. On the other hand, he says (p. 443): "One's first reaction may be that explaining portfolios sorted on the basis of size and book/market, by [grouped] factors sorted on the same basis, is a tautology." Furthermore, he asserts (p.443) "in testing a model, it is exactly the right thing to do to sort stocks into portfolios [groups] based on characteristics related to expected returns." With appropriate research design, econometric model testing results are invariant to sample sort order, but they are not invariant to sample grouping. Grouped samples must be formed from previously sorted samples, but sorted samples do not

require grouping. Group-based factors indirectly entail the variables used as the basis to sort the sample and then partition the sample into groups. Thus, size- and value-related group-based factors are price- and shares-entailing group-based factors.

More to the point, in testing a *causal, inferential, econometric, linear regression* model *of return for stock-portfolio pricing*, it is exactly the right thing to do to sort stocks into portfolios based on characteristics *if and only if the characteristics are not linearly* related to expected returns. It is a logically valid procedure for Fama-MacBeth tests of the CAPM and thus is an acceptable thing to do. In contrast, it is not a logically valid procedure for Fama-French tests of the Three-Factor Model and thus is exactly the wrong thing to do. The reference to CAPM market-beta factor is beside the point, because market-beta, the explanatory variable, is not linearly related to expected return, the dependent variable.

The example of common co-movement of an alphabet-based variable also is beside the point, because it ignores the fact of tautology and vicious circular reasoning. A factor that is linearly related to return, such as a price-, shares- or dividends-entailing size-related or value-related factor, will necessarily co-move with price-, shares- and dividends-entailing return. After more than 20 years and more than 20 published articles about stock pricing in academic journals, Messrs. Fama and French still have not found a valid source of common movement in stock returns.

Circular single-equation simultaneity can be introduced into the testing of an econometric model either directly or indirectly. Direct circular single-equation simultaneity occurs when the logically circular, alleged explanatory factor is specified in the model equation. Indirect circular single-equation simultaneity occurs when the

logically circular factor is used to sort and partition the sample into groups or portfolios formed on the logically circular factor for the purpose of constructing a group-based or portfolio-based explanatory factor that is specified in the model equation.

In testing a casual, inferential, econometric model of expected return for stock-portfolio pricing, it is generally accepted standard scientific research methodological practice to specify a group-based explanatory factor in a model equation, *if and only if* (i) the variables used for sorting and partitioning the sample to construct the group-based factor *are not* linearly related to price-, shares- and dividends-entailing expected total return, and they are reported to be independent factors with independent observations and no adverse implications concerning causal inference; or (ii) the variables used for sorting and partitioning the sample to construct the group-based factor *are* linearly related to price-, shares- and dividends-entailing expected total return, and they are reported to be non-independent factors with non-independent observations that seriously violate the Gauss-Markov Theorem, where applicable, and in all cases, they result in a serious violation of genuine method in the form of the fatal fallacy known in classical logic as vicious circular reasoning, and thus they have serious implications that invalidate logical inference.

If stock portfolios have no spread in average returns—if they are just choosen as random portfolios to test an econometric model of return, then there may be insufficient variation for tests of the asset pricing model to produce statistically significant results. Valid scientific research is not easy. In the case of variable with a low range of variation, it may be helpful to increase the sample size where possible, or change the number of groups, or forgo the construction of group-based factors altogether.

Sorting and grouping a sample is a technique of succinctly presenting certain kinds of data for analysis. Sorting and grouping for purposes of model testing, rather than for purposes of data presentation, is a matter of proper research design and methodology. In some research settings, an untreated control group and a treated experimental group are required to isolate certain identifiable threats to validity. For more information about groups in research methodology, see *Quasi-Experimentation: Design and Analysis Issues for Field Settings*, Thomas D. Cook and Donald T. Campbell, 1979, Houghton Mifflin. But sorting and grouping for purposes of constructing group-based dependent and explanatory variables can indirectly introduce tautologies in the form of logically circular single-equation simultaneities into an econometric model equation.

Partitioning or grouping a sample is generally not required for testing econometric models, except where dummy variables are used. A factor or characteristic that works for a sample of groups or portfolios of stocks will work better for a sample of individual stocks for two reasons: first, grouping results in a loss of information; and second, grouping introduces new information based on variable used for grouping. When a sample is sorted and grouped before testing an econometric model, the sorting and grouping variable and its full implications concerning the validity and bias of test results should be reported. Grouped samples of stocks in portfolios, the number of portfolios, and the cutting points between portfolios, are arbitrary decisions in the sense that one choice is not inherently better than other choices. Most importantly, if sorting and grouping a sample cannot be done without introducing fatal fallacies and serious biases that invalidate logical inference, then the sample should not be sorted and grouped before testing an econometric model.

The argument over the logical and scientific *validity* of the size and book-to-market value factors in an econometric model of return is not a topic of fair, open discussion in academic finance. Messrs. Fama and French and their collaborators are silent about the argument, and they apparently seek to limit discussion about the Three-Factor Model of expected return for stock-portfolio pricing to the unscientific topics of "refinements and interpretations" of the atheoretic, *ad hoc*, logically viciously circular, fatally fallacious, size- and value-related alleged explanatory factors. Mr. Cochrane is silent about how one could refine the econometrically non-refineable and interpret the logically non-interpretable.

The false and misleading discussion of the Three-Factor Model of return and its alleged size- and value-related explanatory factors is transparently tendentious. Mr. Cochrane is motivated in his attempt to justify the unjustifiable, excuse the inexcusable, and make appear conscionable the unconscionable. He has had bias-inducing ties and conflicts of interest at least since 2001 as a member of the board of directors of the Center for Research in Security Prices (CRSP) at the Graduate School of Business at the University of Chicago. CRSP is headed by Eugene F. Fama, who is also a stockholder in privately owned Dimensional Fund Advisors (DFA), a member of the board of directors of DFA, and director of research for DFA. DFA, an investment advisory and index fund management firm, offers about 40 index funds. DFA stock equity index funds are based on the Fama-French Three-Factor Model of return and its size- and value-related factors.

Appendix: Three-Factor Model Equations

Fama-French Three-Factor Model of expected total return for stock-portfolio pricing

1. Fama and French (1992, *Journal of Finance*, pp. 429, 430, 431, 438-439, 448)

Part I. Preliminaries. *A. Data*

We use all nonfinancial firms in the intersection of (a) the NYSE, AMEX, and NASDAQ return files from the Center for Research in Security Prices (CRSP) and (b) the merged COMPUSTAT annual industrial files of income-statement and balance-sheet data, also maintained by CRSP.

Part I. Preliminaries. *Section B. Estimating Market βs*

Our asset-pricing tests use the cross-sectional regression approach of Fama and MacBeth (1973). Each month the cross-section of returns on stocks is regressed on variables hypothesized to explain expected returns. Since size (ME), E/P, leverage, and BE/ME are measured precisely for individual stocks, there is no reason to smear the information in these variables by using portfolios as in the Fama-MacBeth regressions. Most previous tests use portfolios because estimates of market betas are more precise for portfolios. Our approach is to estimate βs for portfolios and then assign a portfolio's β to each stock in the portfolio. This allows us to use individual stocks in the Fama-MacBeth asset-pricing tests.

Part I. Preliminaries *Section B.1. β Estimation: Details*

To allow for variation in β that is unrelated to size, we subdivide each size decile into 10 portfolios on the basis of pre-ranking βs for individual stocks.

After assigning firms to the size-β portfolios in June, we calculate the equal-weighted monthly returns on the portfolios for the next 12 months, from July to June. In the end, we have post-ranking monthly returns for July 1963 to December 1990 on 100 portfolios formed on size and pre-ranking βs.

Part II. β and Size. *Section B. Fama-MacBeth Regressions*

Table III shows time-series averages of the slopes from the month-by-month Fama-MacBeth (FM) regressions of the cross-section of stock returns on size, β, and the other variables (leverage, E/P, and book-to-market equity) used to explain average returns. [Leverage is measured by ln(A/ME) and by ln(A/BE), where A is total book assets, BE is book value of common equity plus balance-sheet deferred taxes, and ME is market equity (price multiplied by shares).]

Table III (page 439): Average Slopes (*t*-Statistics) from Month-by-Month Regressions of Stock Returns on Beta, Size, Book-to-Market Equity, Leverage, and E/P: July 1963 to December 1990

Stocks are assigned the post-ranking β of the size- β portfolio they are in at the end of June of year *t* (Table I). BE is the book value of common equity plus balance-sheet

deferred taxes. A is total book assets, and E is earnings (income before extraordinary items, plus income-statement deferred taxes, minus preferred dividends). BE, A, and E are for each firm's latest fiscal year ending in calendar year $t - 1$. Firm size ln(ME) is measured in June of year t. In the regressions, these values of the explanatory variables for individual stocks are matched with CRSP returns for the months from July of year t to June of year $t + 1$. The gap between the [Compustat annual] accounting data and the [CRSP calculated monthly] returns ensures that the accounting data are available prior to the returns. If earnings are positive, E(+)/P is the ratio of total earnings to market equity and E/P dummy is 0. If earnings are negative, E(+)/P is 0 and E/P dummy is 1.

The average slope is the time-series average of the monthly regression slopes for July 1963 to December 1990, and the t-statistic is the average slope divided by its time-series standard error.

On average, there are 2267 stocks in the monthly regressions. To avoid giving extreme observations heavy weight in the regressions, the smallest and largest 0.5% of the observations (11.3 stocks) on E(+)/P, BE/ME, A/ME, and A/BE are set equal to the next largest or smallest values of the ratios (the 0.005 and 0.995 fractiles). This has no effect on inferences.

Note: The following return model equation is based on all column headings in Table III:

$$R = a + b_1\beta + b_2\ln(ME) + b_3\ln(BE/ME) + b_4\ln(A/ME) + b_5\ln(A/BE) + b_6(E/P\ Dummy) + b_7(E(+)/P) + e$$

The different model specifications with expected return, R, as the left-hand-side variable include the following combinations of right-hand-side variables:
01. β;
02. ln(ME);
03. β and ln(ME);
04. ln(BE/ME);
05. ln(A/ME) and ln(A/BE);
06. E/P Dummy and E(+)/P;
07. ln(ME) and ln(BE/ME);
08. ln(ME), ln(A/ME) and ln(A/BE);
09. ln(ME), E/P Dummy and E(+)/P;
10. ln(ME), ln(BE/ME), E/P Dummy and E(+)/P;
11. ln(ME), ln(A/ME), ln(A/BE), E/P Dummy and E(+)/P.

Part IV. A Parsimonious Model for Average Returns *Section C. Subperiod Averages of the FM Slopes*

Table VI shows the average FM slopes for two roughly equal subperiods … from two regressions. For perspective, average returns on the value-weighted and equal-weighted (VW and EW) portfolios of NYSE stocks are also shown.

Table VI (page 448): Subperiod Average Monthly Returns on the NYSE Equal-Weighted and Value-Weighted Portfolios and Subperiod Slopes from the Monthly FM Cross-Sectional Regressions of Returns on (a) Size (ln(ME)) and Book-to-Market Equity (ln(BE/ME)), and (b) β, ln(ME), and ln (BE/ME)

Mean is the time-series mean of a monthly return, Std is its time-series standard deviation, and *t*(Mn) is Mean divided by its time-series standard error.

$R_{it} = a + b_{2t}\ln(ME_{it}) + b_{3t}\ln(BE/ME_{it}) = e_{it}$

$R_{it} = a + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + b_{3t}\ln(BE/ME_{it}) = e_{it}$

2. Fama, French, Booth and Sinquefield (1993, *Financial Analysts Journal*, p. 39)

Table II. Means, Standard Deviations (Std.) and Correlations for the Monthly Explanatory Returns (%, annualized) in the Three-Factor Time-Series Regressions 7/73-12/91 (222 monthly observations)

We construct six portfolios (S/L, S/M, S/H, B/L, B/M, B/H) from the intersections of the two ME and the three BE/ME groups. … Monthly **value-weighted returns** on the six portfolios are calculated from July of year t to June of t + 1, and the portfolios are reformed in June of t + 1 [bold emphasis in the original].

RM is the return on the value-weighted portfolio of stocks in the six size-BE/ME portfolios, plus the negative-BE stocks excluded from the portfolios. RF is the one-month Treasury bill rate, observed at the beginning of the month. RM-RF is our proxy for the market factor in stock returns.

A Three-Factor Story. Fama and French find that a **three-factor** asset-pricing **model** explains the average returns on NYSE, AMEX and NASD stocks. In their model, a security's expected return is determined by its sensitivities to an overall market risk factor and risk factors related to size and book-to-market-equity. The sensitivities are estimated by regressing the time series of the security's returns on proxies for the common risk factors [bold emphasis in the original]:

Eq. 1. $R_i(t) - RF(t) = a_i + b_i[RM(t) - RF(t)] + s_iSMB(t) + h_iHML(t) + e_i(t)$.

In this regression, $R_i(t) - RF(t)$ is the return on asset i in excess of the risk-free rate (the one-month Treasury bill rate) for month t; $RM(t) - RF(t)$ is the excess return for month t on a value-weighted market portfolio of stocks; and SMB(t) and HML(t) are the returns on portfolios constructed to mimic size and book-to-market-equity risk factors in returns.

Table III. Regressions of Value-Weighted Monthly [Small to Big] Quintile Returns on Proxies for Market (RM-RF), Size (SMB) and Book-to-Market (HML) Risk Factors in Returns, July 1973-December 1991 (222 months)

$R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB + hHML + e$

NYSE Quintile Returns (Small to Big)
NASD Quintile Returns (Small to Big)
Difference between NYSE Quintile and NASD Quintile Returns (Small to Big)

3. Fama and French (1993, *Journal of Financial Economics*, pp. 8-13, 16, 22-25, 46-48, 50-51)

2.1.2. *Stock-market factors*

The Building Blocks – To study economic fundamentals, Fama and French (1992b, working paper; *Proceedings: Seminar on the Analysis of Security Prices,* CRSP) use six portfolios formed from sorts of stocks on *ME* and *BE/ME*. We use the same six portfolios here to form portfolios meant to mimic the underlying risk factors in returns related to size and book-to-market equity.

In June of each year *t* from 1963 to 1991, all NYSE stocks on CRSP are ranked on size (price times shares). The median NYSE size is then used to split NYSE, Amex, and (after 1972) NASDAQ stocks into two groups, small and big (*S* and *B*). Most Amex and NASDAQ stocks are smaller than the NYSE median, so the small group contains a disproportionate number of stocks (3,616 out of 4,797 in 1991). Despite its large number of stocks, the small group contains far less than half (about 8% in 1991) of the combined value of the two size groups.

We also break NYSE, Amex, and NASDAQ stocks into three book-to-market equity groups based on the breakpoints for the bottom 30% (*Low*), middle 40% (*Medium*), and top 30% (*High*) of the ranked values of *BE/ME* for NYSE stocks. We define book common equity, *BE*, as the COMPUSTAT book value of stockholders' equity, plus balance-sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. … Book-to-market equity, *BE/ME*, is then book common equity for the fiscal year ending in calendar year $t - 1$, divided by market equity at the end of December of $t - 1$. We do not use negative-*BE* firms, which are rare before 1980, when calculating the breakpoints for *BE/ME* or when forming the size-*BE/ME* portfolios.

Size – Thus, *SMB* is the difference between returns on [three] small- and [three] big-stock portfolios with about the same weighted-average book-to-market equity.

BE/ME – *HML* is the difference, each month, between the simple average of the returns on the two high-*BE/ME* portfolios (*S/H* and *B/H*) and the average of the returns on the two low-*BE/ME* portfolios (*S/L* and *B/L*).

Market – Finally, our proxy for the market factor in stock returns is the excess market return, *RM-RF*. *RM* is the return on the value-weighted portfolio of the stocks in the six size-*BE/ME* portfolios, plus the negative-*BE* stocks excluded from the portfolios. *RF* is the one-month bill rate.

2.2. *The returns to be explained*

Stocks – For stocks, we use excess returns on 25 portfolios, formed on size and book-to-market equity, as dependent variables in the time-series regressions.

Table 1 (page 11): Descriptive statistics for 25 stock portfolios formed on size and book-to-market equity, 1963-1991, 29 years.

The 25 size-*BE/ME* stock portfolios are formed as follows. … The 25 size-*BE/ME* portfolios are formed as the intersections of the five size and the five *BE/ME* groups. …

The descriptive statistics are computed when the portfolio is formed in June of each year, 1963-1991, and are then averaged across the 29 years.

3. *The playing field*

Table 2 summaries the dependent and explanatory returns in the time-series regressions. The average excess returns on the portfolios that serve as dependent variables give perspective on the range of average returns that competing sets of risk factors must explain. The average returns on the explanatory portfolios are the average premiums per unit of risk (regression slope) for the candidate common risk factors in returns.

3.1. The dependent returns

Stocks – The 25 stock portfolios formed on size and book-to-market equity produce a wide range of average excess returns. Our time-series regressions attempt to explain the cross-section of average returns with the premiums for the common risk factors in returns. The wide range of average returns on the 25 stock portfolios, and the size and book-to-market effects in average returns, present interesting challenges for competing sets of risk factors.

3.2. The explanatory returns

In the time-series regression approach to asset-pricing tests, the average risk premiums for the common factors in returns are just the average values of the explanatory variables.

Page 16: We turn now to the asset-pricing tests. The three stock-market returns, *RM-RF*, *SMB* and *HML*, are risk factors in the sense that they capture common (shared and thus undiversifiable) variation in stock returns.

Table 5 (pp. 22-23): Regressions of excess stock and bond returns (in percent) on the mimicking returns for the size (*SMB*) and book-to-market equity (*HML*) factors: July 1963 to December 1991, 342 months.

$$R(t) - RF(t) = a + sSMB(t) + hHML(t) + e(t)$$

Dependent variable: Excess returns on 25 stock portfolios formed on size and book-to-market equity

SMB (small minus big), the return on the mimicking portfolio for the common size factor in stock returns, is the difference each month between the simple average of the percent returns on the three small-stock portfolios (*S/L*, *S/M*, and *S/H*) and the simple average of the returns on the three big-stock portfolios (*B/L*, *B/M*, and *B/H*). *HML* (high minus low), the return on the mimicking portfolio for the common book-to-market equity

factor in returns, is the difference each month between the simple average of the returns the two high-*BE/ME* portfolios (*S/H* and *B/H*) and the average of the returns on the two low-*BE/ME* portfolios (*S/L* and *B/L*). [There are six size-*BE/ME* portfolios formed by the intersections of two size fractiles, *S* and *B*, and three *BE/ME* fractiles, *L*, *M* and *H*.]

Table 6 (pp. 24-25): Regressions of excess stock and bond returns (in percent) on the excess market return (*RM-RF*) and the mimicking returns for the size (*SMB*) and book-to-market equity (*HML*) factors: July 1963 to December 1992, 342 months (*t* indexes time in months).

$$R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) + e(t)$$

Dependent variable: Excess returns on 25 stock portfolios formed on size and book-to-market equity [the intersections of size (ME) quintiles and book-to-market equity (BE/ME) quintiles.]

RM is the value-weighted percent monthly return on all the stocks in the 25 size-*BE/ME* portfolios, plus the negative–*BE* stocks excluded from the 25 portfolios. *RF* is the one-month Treasury bill rate, observed at the beginning of the month. *SMB* (small minus big) is the return on the mimicking portfolio for the size factor in stock returns. *HML* (high minus low) is the return on the mimicking portfolio for the book-to-market factors. (See table 5.) … The 25 size-*BE/ME* stock portfolios are formed as follows. Each year *t* from 1963 to 1991 NYSE quintile breakpoints for size, *ME*, measured at the end of June, are used to allocate NYSE, Amex, and NASDAQ stocks to five size quintiles. Similarly, NYSE quintile breakpoints for *BE/ME* are used to allocate NYSE, Amex, and NASDAQ stocks to five book-to-market equity quintiles. In *BE/ME*, *BE* is book common equity for the fiscal year ending in calendar year $t - 1$, and *ME* is for the end of December of $t - 1$. The 25 size-*BE/ME* portfolios are the intersections of the five size and the five *BE/ME* groups. Value-weighted monthly percent returns on the 25 portfolios are calculated from July of *t* to June of $t + 1$.

COMMENTS

In Fama and French (1992), the Fama-French Three-Factor Model of return is direct circular single-equation simultaneity due to specification in the model equation of size, *BE/ME*, *A/ME* or *E/P*, all of which entail the price variable and the shares variable that also are entailed by the dependent variable, expected total return.

In Fama and French (1993), the Fama-French split-sample *ad hoc* diagnostic test of the independence of the size- and value-related factors in the Fama-French Three-Factor Model of return is indirect circular single-equation simultaneity due to sorting and grouping of the sample data on price- and shares-entailing size and *BE/ME* to construct portfolio-based factors.