

Fatal Fallacy: Summary

The fatal fallacy in the Fama-French Three-Factor (FF3F) stock portfolio pricing model of return is the logical fallacy of vicious circular reasoning. It is known in econometrics as a simultaneity. Technically speaking, it is a logically circular type of single-equation simultaneity. The fallacy of circular simultaneity is fatal in the sense that it is irremediable, irrefutable, terminal, not logically valid and not scientifically valid.

Logic, algebra and econometrics share the principle of true method. True method means proceeding from the better known to the lesser known, not from the known to the equally known, which is the fallacy of circular reasoning. Basic logic, beginning algebra and introductory econometrics are equivalent ways to explain the fatal fallacy of the FF3F model. The fallacy of circular reasoning, when expressed in terms of econometrics, is circular simultaneity. Non-isolation of the left-hand side of an algebraic equation, when expressed in terms of econometrics, is circular simultaneity.

Logic	Fallacy of circular reasoning or “begging the question”
Algebra	Non-isolation of left-hand side of equation
Econometrics	Simultaneity (logically circular type)

It can sometimes be quite difficult to detect circular reasoning in an argument; but once revealed, the logical circularity between premiss and conclusion is not difficult to understand. Similarly, it can sometimes be difficult to detect the same variable on both sides of an algebraic equation, especially when it is part of another variable; but once revealed, the failure to isolate the left-hand side of the equation is not difficult to understand. Likewise, it can sometimes be difficult to detect circular simultaneity, especially when the same variable is embedded in the dependent and an explanatory variable of a model; but once revealed, the simultaneity is not difficult to understand.

Rational Thought System	A	B	A and B are Independent?
True Method	Better known	Lesser known	Yes
Scientific Method	Observations	Prediction	Yes
Logical Argument	Premises	Conclusion	Yes
Algebraic Equation	RHS terms	LHS term	Yes
Econometric Model	Explanatory variables	Dependent variable	Yes
Stock Pricing Model	Risk factors	Return	Yes

In a rational thought system, the movement is from A to B, and A and B must be independent of one another. Any violation of these requirements is arational. This is the logic of discovery that was invented by the ancient Greeks and perfected over time.

Someone who understands introductory econometrics should understand a circular simultaneity once it has been revealed in a stochastic model. A circular simultaneity is fallacious, meaningless, non-interpretable, indeterminate, irrational, inefficient, economically wasteful, and not scientifically valid. And most are fatally fallacious.

Being published at all or being published in any particular scientific research journal is incidental to the facts and logic of a logical argument. Many empirical articles published in prominent academic research journals such as the *Journal of Finance* and the *Journal of Financial Economics* are neither scientifically interesting nor important because they violate true method or, more particularly, they are based on circular reasoning in the form of logically circular econometric simultaneity. This is the case with the FF3F model.

Publication adds nothing to the facts and logic of a case. Publication is no substitute for accurate facts and valid logic. Publication per se in any medium is no guarantee of understanding of the truth of a matter. Publication serves to establish precedence of discovery and to make new knowledge more accessible to others.

Formal education in financial economics or finance gives a person an investment to protect. Such education can introduce bias in thinking, even if only on a subconscious level. As Francis Bacon explained four centuries ago, these two phenomena are characteristic of all fields of knowledge. Thus the persons best able to understand the FF3F model fallacy without potential bias are those who understand econometrics and who have no vested interest in financial economics or finance.

The FF3F Model

The FF3F model can be described in words. The left-hand side (LHS) of the model equation consists of the dependent variable, which is return. The operational definition of return includes price, dividends and shares. The right-hand side (RHS) of the model equation consists of three alleged explanatory variables: a variable directly related to a stock market index, a variable directly related to size (market equity), and a variable directly related to value (book equity to market equity ratio). The operational definition of size and thus of any size-related variable includes price and shares. The operational definition of value and thus of any value-related variable includes price and shares.

The fatal fallacy of the FF3F model also can be described in words. If the same variable appears on both sides of the equation for an econometric model, it is a circular simultaneity. The variable may be specified in a model explicitly by itself, such as the variable named return; or the variable may be specified in a model through the operational definition of another variable, such as the variable named price that is part of the return variable, of the size-related variable and of the value-related variable. Lastly, the variable may be introduced indirectly into the testing and estimating of an econometric model, and this occurs if the data is sorted by the variable.

The return variable on the LHS and a size-related variable on the RHS both include price, and this combination creates a circular simultaneity. The return and size-related variable

also include shares, so again this creates a circular simultaneity. Likewise, the return variable on the LHS and a value-related variable on the RHS both include price, and this combination creates a circular simultaneity. The return and value-related variable also include shares, so again this creates a circular simultaneity. Each of these four combinations is sufficient to result in a circular simultaneity. The market-related variable in FF3F can be operationally defined so as to avoid circular simultaneity.

It is important to not be distracted by the names of variables, but rather to look closely at the operational definitions of every variable specified in an econometric model and of any variable that may be used to sort the sample used in testing and estimating an econometric model. A sorted sample is not a randomly drawn sample.

Because of the logical fallacy of circular reasoning, algebraic non-isolation of the left-hand side, and econometric circular simultaneity, the FF3F model cannot be truly represented as being scientifically valid. It cannot be truly represented as being designed to earn consistent long-term average risk-adjusted expected returns that are higher than conventional market benchmarks. Such representations are meaningless at best.

The fatal fallacy can be expressed in math also. An abstract simplified totally deterministic (non-stochastic) version of the Fama-French Three-Factor Model will serve the purpose of explaining the fallacy.

$$R = M + S + V$$

where R is expected Return, the M term includes the level of the stock Market or a factor directly related to the level of the stock Market, the S term includes Size or a factor that is directly related to Size as one of its parts, and the V term includes Value or a factor directly related to Value as one of its parts.

We need the operational definitions of each of the four variables in the model equation, expressed in terms of their ultimate irreducible component variables. The operational definition of Return for a common stock includes Price (P), Number of shares outstanding (N), and Dividends (D).

The operational definition of the Market risk factor is a broad market proxy such as the Wilshire 5000 in the U.S. When a particular common stock is specified on the left-hand side of the model equation, the same common stock can be omitted from the proxy if it is included in the proxy definition, and this avoids the inclusion of the same variable on both sides of the model equation at the same time.

The operational definition of Size is market value of equity capitalization, and it is equal to Price (P) multiplied by Number of shares outstanding (N). The operational definition of Value is Book Equity to Market Equity Ratio, and it is equal to Book Value of Equity divided by Market Value of Equity or Size ($P \times N$).

$$\begin{array}{rclclcl}
 \mathbf{R} & = & \mathbf{M} & + & \mathbf{S} & + & \mathbf{V} \\
 \mathbf{P} & = & & & \mathbf{P} & + & \mathbf{P} \\
 \mathbf{N} & = & & & \mathbf{N} & + & \mathbf{N} \\
 \mathbf{D} & & & & & &
 \end{array}$$

One diagnostic key to assessing the scientific validity of an econometric model is the detection of any simultaneity, whether an isolated type or a circular type. A logically circular type of single-equation simultaneity occurs when the same variable is included on both the left-hand side and the right-hand side of the model equation at the same time. It can be observed that there are four simultaneities embedded in the Three-Factor Model: (1) *Price* embedded in both *Size* and *Return*; (2) *Number of shares* embedded in both *Size* and *Return*; (3) *Price* embedded in both *Value* and *Return*; (4) *Number of shares* embedded in both *Value* and *Return*.

The following keywords indicate concepts that are useful for understanding the Three-Factor Model for stock pricing: capital asset pricing, circular reasoning, econometric model (stochastic or probabilistic), fatal fallacy, financial assets, market efficiency, opportunity cost, portfolio pricing (versus individual stocks), rational pricing, rational expectations, return (rate of), scientific logic, scientific method, scientific validity, simultaneity, and true method.

For many persons the most problematic of these concepts are the fallacy of circular reasoning and probabilistic models, both of which can be subtle yet can appear to be deceptively easy to understand. Persons with Ph.D. degrees and high measured IQs often do not correctly comprehend either of these two concepts. Yet they think they understand them, and this blocks open-mindedness and learning. When asked, they say that they understand these concepts and sincerely believe what they say.

The difficulty of understanding probability and uncertainty is mentioned by James Gleick in his 1992 book entitled *Genius: The Life and Science of Richard Feynman*, on pages 373 to 375. Mr. Feynman reportedly was aware of the naiveté of his professional scientific colleagues on the subject of probability and chance occurrence.

Similar naiveté exists for the subject of circular reasoning, which can be quite difficult to detect. For many persons, once it is pointed out, he or she still does not understand why circular reasoning is unacceptable for scientific and logical reasoning, nor what the full implications are for circular reasoning. The detection of circular reasoning and the application of probabilistic thinking have many practical applications in everyday life, including judgments about investment opportunities.

The FF3F model is thoroughly analyzed in the article “Asset Pricing Simultaneity, Three-Factor Model and Cost Analysis”, Robert D. Coleman, *Indian Journal of Economics and Business*, Vol. 4, No. 1, (June, 2005): 73-94 <<http://www.ijeb.com/journals/index.html>>. The article is in two parts. The first part demonstrates that each of the arguments in the financial economics literature in support of the FF3F model is not scientifically valid. The FF3F model is shown to be a theoretical failure, a methodological failure and an

empirical failure. When correctly isolated to avoid logical circularity, the FF3F model is quadratic rather than linear in form. The FF3F model is market timing in disguise.

The second part of the article estimates the cost of the fatal fallacy to investors. A typical retirement-savings plan participant loses an estimated \$20,000 due to excess asset management fees on size-related and value-related index mutual funds. The total of these excess fees is estimated to exceed \$1 billion U.S. dollars a year. In addition, the appendix to the article explains why the FF3F model fallacy is fatal in all circumstances.

An analogy may be useful. A highly successful blockbuster drug [FF3F model] on the market is approved by the U.S. FDA and shown in the professional literature to be beneficial, but it is actually a placebo at best that continues to needlessly cost patients vast sums of excess money annually. The blockbuster drug is claimed by its creators and sellers to outperform in tests the earlier most widely prescribed drug [CAPM]. Medical doctors and their patients need to know reliably and authoritatively about the validity of allegedly scientific double-blind controlled testing [OLS, GMM, Hausman's specification error test modified to test exogeneity, Fama-French Split-Sample *ad hoc* diagnostic test of FF3F model, etc.] of the blockbuster drug. Biased tests lead to unsafe harmful drugs. The main points of the pharmaceutical analogy are first, what is true, and second, who benefits and who pays the cost.

Another analogy may be helpful. Two professors, Tama and Trench, published their novel research findings about shoes sizing. They found that expected shoe size (ESS) is determined by three variables: foot opposite (FO), foot length (FL) and foot width (FW). The robust research included a sample of observations from all over the world. Their result was found to be invariant to age, gender, height, weight, race, religion and nationality. The rigorous field research was done by the professors who personally measured the length and width of both the right foot and the left foot of the volunteers in the study. In addition, the professors verified the sizes of all shoes of the volunteers with their original manufacturers.

The resulting Tama-Trench Three-Factor (TT3F) model in math is expressed as $ESS = FO + FL + FW$. The TT3F model was adopted by academia and the footwear industry as a major empirical advance in the knowledge of shoe-sizing, which is beneficial because it increases comfort, improves health and reduces waste. The professors reported that the foot opposite (FO) variable had the greatest explanatory power of the three explanatory variables in their statistical model using Ordinary Least Squares estimation methodology and classical linear regression theory. The FO variable is equal to the shoe size of the left foot, and by convention ESS is equal to the shoe size of the right foot. The FO variable had such high explanatory power, with R-squared above 98%, that it was dubbed the "size effect". The TT3F logo is licensed to footwear manufacturers that advertise the highest quality of premium-fitting shoes sold at premium prices. The TT3F shoe-sizing model is absurd and ridiculous, just like the FF3F stock pricing model, and both for the same reasons. The main points of the shoe-sizing analogy are first, the difference between deterministic non-causal relationships and stochastic causal relationships, and second, the fallacy of circular reasoning.

Conclusions and Implications

The FF3F model is not scientifically valid because it commits the fallacy of circular reasoning. The Fama-French Split-Sample (FFSS) *ad hoc* diagnostic test is alleged to test the independence of the size and value factors in the FF3F model. The FFSS test itself is not scientifically valid because it also commits the fallacy of circular reasoning. On scientific grounds, the size factor, the value factor, and the FF3F model must be rejected.

Investors who uncritically accept the FF3F model and the related investment strategies must not understand the fatal fallacy of circular reasoning. Such investors are like docile sheep that willingly go to be fleeced while insisting they are going to be groomed.

What are the investment strategies that logically follow from the FF3F model? One FF3F investment strategy is based on the size factor. Small-cap stocks are alleged to earn superior long-term average risk-adjusted returns. Another FF3F investment strategy is based on the value factor. Value-style or low book-to-market equity ratio stocks, in contrast to growth-style stocks, are alleged to earn superior long-term average risk-adjusted returns. A third FF3F investment strategy is based on the combination of the size and value factors. Small-cap value-style stocks are alleged to earn superior long-term average risk-adjusted returns. None of these three strategies is scientifically valid because each is based on the fallacy of circular reasoning and on the failure to properly account for probability.

Investment strategies that sector the market on the basis of market capitalization, book-to-market equity ratio, earnings/price ratio or any other price ratio are a simplistic form of market timing. One alternative strategy to earn the highest long-term average risk-adjusted return is to invest in the lowest-cost total stock market index mutual fund.