

Visual Detection of Circular Reasoning: Three-Factor Model

Circular reasoning is a logical fallacy that takes as a premise the very thing that is to be proved in the conclusion. A logical fallacy is illogical and erroneous. Circular reasoning takes the form: In other words they are trying to tell us that X is true because X is true! Circular reasoning is meaningless. Circular reasoning is nonsensical. Circular reasoning is not interpretable. Circular reasoning is not scientifically valid.

The fallacy of circular reasoning is discussed in books on basic logic. The fallacy of circular reasoning was called “begging the question” by Aristotle in his book on propositional logic. Information about logical fallacies and circular reasoning can be found on the Internet with a search using the key word “fallacy”.

In beginning algebra the same principle is expressed in the rule that you must isolate the left-hand side of an equation before you can solve it, without resorting to guessing the answer.

In introductory econometrics, the same principle is expressed in the diagnosis of something known as a single-equation simultaneity or simply as a simultaneity. More particularly, it is a logically circular form of single-equation simultaneity and can be called a circular simultaneity. A circular simultaneity occurs whenever the identical variable appears at the same time on both sides of a model equation.

To understand the Fama-French Three-Factor (FF3F) model of return for stock portfolio pricing in its true light, it is necessary to first understand circular reasoning. The left-hand side (LHS) of the FF3F model equation is Expected Return. The component variables of Expected Return are Beginning Price, Ending Price, Beginning Shares, Ending Shares, and Dividends.

The Size-related risk factor on the right-hand side (RHS) of the FF3F model equation has two component variables. They are Beginning Price and Beginning Shares. The Value-related risk factor on the RHS of the FF3F model equation has three component variables. They are Book Equity, Beginning Price and Beginning Shares.

The simultaneous presence of Beginning Price in Expected Return on the LHS and in the Size risk factor on the RHS is an instance of a circular simultaneity. Likewise, the simultaneous presence of Beginning Price in Expected Return on the LHS and in the Value risk factor on the RHS is an instance of a circular simultaneity.

Therefore, Size must be rejected from a return model. Likewise, Value must be rejected from a return model. Once seen in their true light, the FF3F model and the Fama-French Split-Sample (FFSS) econometric diagnostic test of the FF3F model are stunning in their audacity and deceptiveness. How could this common fallacy in the FF3F model and in the FFSS test escape detection or not be published in any finance or financial economics peer-reviewed scientific research journal for so many years?

Visual Diagnostic Test for Circular Reasoning

The FF3F model can be described in math, in words and in images. Visualization techniques using graphic images can be used to diagnose the specification of an econometric model to determine whether or not there is any circular reasoning among the specified variables. The idea is to picture each side of a model equation as a strand of colored beads and then compare the two strands.

Imagine a supply of beads. Each bead has a protuberance on one side and a recess on the opposite side. By inserting the protuberance of one bead into the recess of another bead, the beads can be strung together into a strand without the use of string.

The bead colors are white, black, and the seven colors of the visible light spectrum. Dispersion of visible light through a prism produces the colors red (R), orange (O), yellow (Y), green (G), blue (B), indigo (I), and violet (V). It is because of this that visible light is sometimes referred to as ROY G. BIV. The seven colors appear in standard hue, light hue and dark hue, resulting in 21 available colors, in addition to white and black.

A test of the FF3F model uses eight colors:  .

Each side of the model equation is represented as a strand of beads. The left-hand side (LHS) becomes the left-hand strand, and the right-hand side (RHS) becomes the right-hand strand. A bead color is assigned to each term, factor or variable in the econometric model equation as needed to capture all the necessary component variables in the operational definitions used in the testing operations and estimating operations.

This visual diagnostic test for the circular reasoning fallacy has four conventions.

One: No color appears in the strand more than once on the LHS, even if it occurs in the model more than once on the LHS. The LHS is a list of the explained variable or its component parts. The LHS is not a frequency distribution.

Two: Each color appears in the strand on the RHS once for each time it occurs in the model on the RHS. The RHS is a frequency distribution of terms and explanatory variables or their component parts. The RHS is not a list.

Three: There is only one white-colored bead, which always represents the Intercept term; and there is only one black-colored bead, which always represents the Upset term (error term or residual term).

Four: The LHS of the model must be fully decomposed to identify all component variables on the LHS. Five: The RHS of the model must be decomposed only so far as to identify any new variables on the RHS that could appear on the LHS.

In the case of the FF3F model, the following assignment is made going from left to right and alphabetically.

LHS of FF3F model:

Expected Return	Composite of Capital Gains and Cash Dividends.
Capital Gains	Composite of Capital Changes and Price Change.
Capital Changes	Composite of Shares Beginning and Shares Ending.
Shares Beginning:	<i>Red.</i>
Shares Ending:	<i>Orange.</i>
Price Change	Composite of Price Beginning and Price Ending.
Price Beginning:	<i>Yellow.</i>
Price Ending:	<i>Green.</i>
Cash Dividends:	<i>Blue.</i>

RHS of FF3F model:

Intercept term:	<i>White.</i>
Market risk factor:	<i>Indigo.</i>
Size risk factor:	Composite of Price Beginning and Shares Beginning.
Price Beginning	<i>Yellow.</i>
Shares Beginning	<i>Red.</i>
Value risk factor:	Composite of Book Equity and Size.
Book Equity:	<i>Violet.</i>
Size variable:	Composite of Price Beginning and Shares Beginning.
Price Beginning	<i>Yellow.</i>
Shares Beginning	<i>Red.</i>
Upset (Error) term:	<i>Black.</i>

Note that the Market risk factor is a composite of Market Proxy Return and Risk-Free Return, but decomposing it into these two components does not change the outcome of the diagnosis. These two components could be represented by an Indigo colored bead for Market Proxy Return and a Light Indigo colored bead for Risk-Free Return.

The LHS of the FF3F model can be visualized as the following strand of colored beads:

LHS: Red-Orange-Yellow-Green-Blue.

LHS: 

The RHS of the FF3F model can be visualized as the following strand of colored beads:

RHS: White-Indigo-[Light Indigo]-Yellow-Red-Violet-Yellow-Red-Black.

RHS: 

To detect the presence of circular reasoning in the specification of the FF3F model, in contrast to circular reasoning in the testing and estimation of the FF3F model, compare

the LHS and the RHS. There is circular reasoning attributable to model specification if any combination of one bead on the LHS and one bead on the RHS is the same color. Every unique pair of LHS-RHS matching-color beads is an instance of circular reasoning in the model specification.

This visual diagnosis of the FF3F model can be made by answering the following questions.

Question 1: How many same-color LHS-RHS bead pairs are there, if any?

Question 2: What color is each of these LHS-RHS bead pairs?

Question 3: What is the diagnostic test result for the FF3F model?

Question 4: What variables and factors are responsible for any circular reasoning?

Answers: 1. Four, 2. Red, Red, Yellow, Yellow, 3. FF3F Model is circular reasoning, 4. Price Beginning (twice), Shares Beginning (twice), Size, Value.